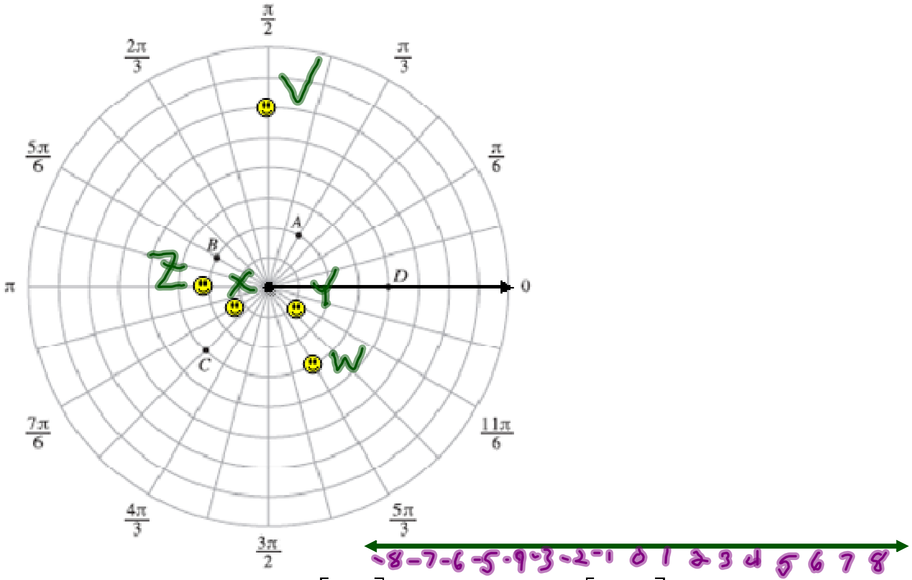
Math 4 Honors
Polar Graphing Group Activity

Name ____________________________
Date ________________________

For years, you have been plotting function equations on a Cartesian coordinate grid. In this activity, you will learn about the polar coordinate system and graphing polar equations. The polar coordinate system consists of a fixed point called the *pole* (similar to the origin in a Cartesian coordinate system) and a fixed ray called the *polar axis*. A polar coordinate graph is usually made on a grid that shows concentric circles that have the pole at the center and rays that begin at the pole and are associated with different amounts of turn. Each point in the polar coordinate system is identified by its distance from the pole, represented by $r$, and an angle of turn from the polar axis, represented by $\theta$. The polar coordinates of a point are given in the form $[r, \theta]$. The coordinate $r$ is called the *radial coordinate* and the coordinate $\theta$ is called the *angular coordinate*. When $\theta$ is positive, the angle is measured in the counterclockwise direction and when $\theta$ is negative, the angle is measured in the clockwise direction.

**I.** On the polar grid below, there are four points that have already been graphed.




**1.** The point $A$ can be represented by the coordinates $\left[2, \frac{\pi}{3}\right]$ or the coordinates $\left[2, \frac{7\pi}{3}\right]$. Explain why these two coordinate pairs represent the same point and give another coordinate pair for this point.

$\frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$ $\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}$ $\left[2, -\frac{5\pi}{3}\right]$

**2.** Point $B$ can be represented by the coordinates $\left[2, \frac{-7\pi}{6}\right]$. Represent point $B$ using a positive value for $\theta$.

$\left[2, \frac{5\pi}{6}\right]$ (one possible answer)

**3.** On a polar graph, a point's distance from the pole will always be a positive number. Because of this, plotting points with a negative $r$ value is not completely straightforward. The point $C$ can be represented by the coordinates $\left[3, \frac{5\pi}{4}\right]$ as well as $\left[-3, \frac{\pi}{4}\right]$. Give **two** different sets of polar coordinates for point D. Make sure one set has a negative radial value.

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$[4, 0]$ $[-4, \pi]$ $[-r, \theta + \pi]$

(possible answers)

**4.** Plot the following points on the polar coordinate grid on the front of this packet. Then provide **two** other coordinate pairs that represent the same point. One of your representations should have a negative radial coordinate and one should have a positive radial coordinate. Additionally, one should have a negative angular coordinate and one should have positive angular coordinate.

(possible answers)

$V\left[6, \frac{\pi}{2}\right] = \left[-6, \frac{3\pi}{2}\right] = \left[6, -\frac{3\pi}{2}\right]$

$W\left[3, \frac{-\pi}{3}\right] = \left[-3, \frac{2\pi}{3}\right] = \left[3, \frac{5\pi}{3}\right]$

$X\left[1.5, \frac{7\pi}{6}\right] = \left[-1.5, \frac{\pi}{6}\right] = \left[1.5, -\frac{5\pi}{6}\right]$

$Y\left[-1, \frac{3\pi}{4}\right] = \left[1, -\frac{\pi}{4}\right] = \left[1, \frac{7\pi}{4}\right]$

$Z[2, 5\pi] = [-2, 0] = [2, \pi]$

**5.** Explain why every point on a polar coordinate system can be identified with an infinite number of coordinates.

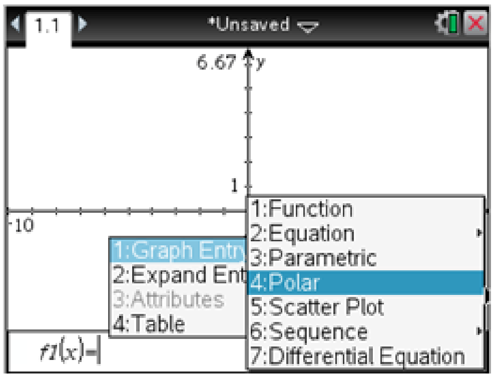
$[r, \theta]$ can be represented by $[r, \theta + 2k\pi]$ k is any integer

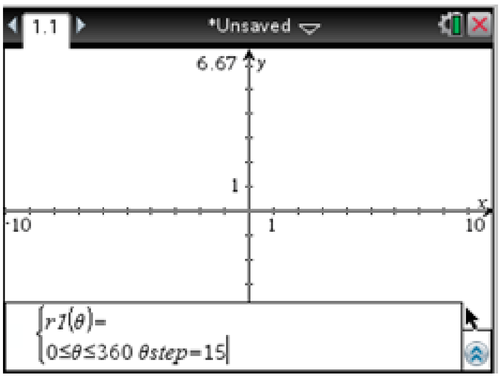
**II.** In the next part of this activity, you will explore graphs of polar equations. A polar equation is a function rule in the form $r = f(\theta)$, where $\theta$ can be measured in radians or degrees.

What is the *independent* variable? $\theta$ What is the *dependent* variable? $r$

***<u>Switch your calculator into degree mode.</u>***

To graph in the polar coordinate system using your calculator, use these settings:



**Use your calculator to explore the following:**

1. Consider equations of the form: $r = a\sin\theta$, $r = a\cos\theta$ Experiment with different values for *a*.

   a. What type of figure is created by these equations? circle

   b. How do the figures differ when different trig functions are used (sin vs. cos)?

   orientation: $\frac{\pi}{2}$ axis polar axis

   c. What is significant about the *a*-value?

   a is the diameter.

2. Consider equations of the form: $r = a \pm b\sin\theta$, $r = a \pm b\cos\theta$ ***Limaçons***

   Graph together: $r = 2+5\sin\theta$, $r = 1+3\cos\theta$ Graph together: $r = 4+3\sin\theta$, $r = 3+2\cos\theta$ Graph together: $r = 4+4\sin\theta$, $r = 2-2\cos\theta$

   a. How do the figures differ when different trig functions are used (sin vs. cos)?

   See 1b.

   b. What is it about the "*a*" & "*b*" values that determines the shape of the graph? (Compare the numbers to each other.)

   $a<b \Rightarrow$ Loop $a>b$ dimpled $a=b$ cardioid

3. Consider equations of the form: $r = a\sin(n\theta)$, $r = a\cos(n\theta)$ ***Rose Curves***

   Graph these functions one at a time: $r = 2\sin(3\theta)$ $r = 4\sin(2\theta)$ $r = 2\cos(3\theta)$ $r = 4\cos(2\theta)$

   a. How do the figures differ when different trig functions are used (sin vs. cos)?

   See 1b.

   b. What determines the length of a petal?

   a = length of petal

   c. What determines the number of petals?

   If n is odd, then n = # of petals.
   If n is even, then 2n = # of petals.

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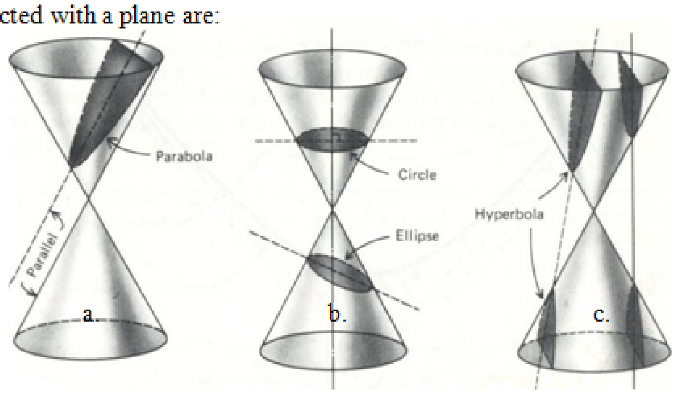
**4.** In mathematics, a **conic section** (or just **conic**) is a curve obtained by intersecting a cone (more precisely, a right circular conical surface) with a plane. The three conic sections that are created when a double cone is intersected with a plane are:

a. Parabola
b. Circle and ellipse
c. Hyperbola




Source: http://project1.caryacademy.org/Earthquakes/2005/HA218/images/conic.jpg

Each of the above curves can be represented in rectangular form as follows:

Circle: $x^2 + y^2 = a^2$ Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Parabola: $y^2 = 4ax$ Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

How might you graph these in your calculator? Think polar! Try this:

Reset the window of your calculator to that of the beginning of part II. Graph each of the following, one at a time.

$$r = \frac{10}{1 + 3\cos\theta} \qquad r = \frac{1}{3 + 2\cos\theta} \qquad r = \frac{1}{2 - 2\cos\theta}$$

(Ignore any lines; they're asymptotes)

a. What is the name of the shape for each figure produced?

hyperbola ellipse parabola

b. How are these equations related to those of the limaçons?

recip. of limagon w/loop recip. of dimp. limagon recip. of cardioid

**5.** Graph each equation below.

a. $r = 10\sin(\theta)\cos^2(\theta)$ This is called a ***Bifolium***. Why?

2 "leaves"

b. $r = \frac{2\theta}{\pi}$ To see this graph better: Change θmax to 3600; then Zoom - Fit.

**FYI** - This called a ***Spiral of Archimedes.***

c. $r = 3\csc(\theta) + 5$ Use the same settings as those from part b.

This is called the ***Conchoid of Nichodemes.*** What happens if this is in terms of the secant?

Turns vertically; oriented to $\frac{\pi}{2}$-axis

**Polar Graphing Practice**

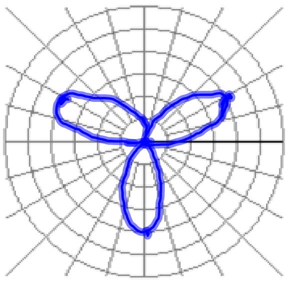
1. What shape is the following graph? $r = 8\sin\theta$ Identify the center and radius. Then convert the equation into rectangular form.
   Recall standard form for the equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$

   Circle
   $(0,4)$
   $r = 4$

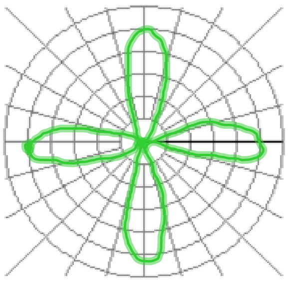
   $x^2 + (y-4)^2 = 16$

2. Sketch accurate graphs of the following. Use a table to help you plot your points.

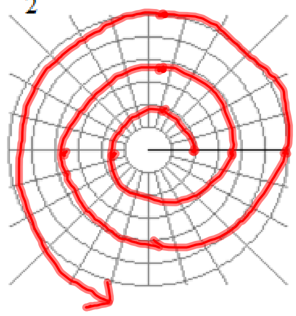
   a. $r = 4\sin(3\theta)$

   b. $r = 5\cos(2\theta)$

   c. $r = \frac{\theta}{2} + 3$ (Graph 2 revolutions.)

**Write polar equations for the following:**

3. A circle with radius 4.8 and oriented to the polar axis $r = 9.6\cos\theta$
4. An example of a hyperbola oriented to the $\pi/2$ axis $r = \frac{5}{1+3\sin\theta}$
5. An example of an ellipse oriented to the polar axis $r = \frac{5}{4+3\sin\theta}$
6. An example of a logarithmic spiral $r = 3^{\theta}$
7. A rose curve with 20 petals of length 13 units, oriented to the $\pi/2$ axis $r = 13\sin(10\theta)$

**Answer the following:**

8. Ralph was confused about how the polar equations above can be considered functions when their graphs do not pass the vertical line test.
   - Use the definition of function to explain why each polar equation ***does*** represent a function.

   For each value of $\theta$, there is only one associated r-value.